

# The Wald entropy formula and loop quantum gravity

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## Abstract

We outline how the Wald entropy formula naturally arises in loop quantum gravity based on recently introduced dimension-independent connection variables. The key observation is that in a loop quantization of a generalized gravity theory, the analog of the area operator turns out to measure, morally speaking, the Wald entropy rather than the area. We discuss the explicit example of (higher-dimensional) Lovelock gravity and comment on recent work on finding the correct numerical prefactor of the entropy by comparing it to a semiclassical effective action.

## 1 Introduction

A key feature that is expected from a theory of quantum gravity is an explanation for the thermodynamic behavior [1] of black holes observed in classical general relativity (GR). By today, several approaches to quantum gravity and semiclassical gravity have addressed this issue and offered different, at times seemingly unrelated, explanations. Moreover, the different approaches are not necessarily applicable to all classes of gravitational theories, such as Lovelock gravity or supergravity, or types of black hole solutions such as extremal or non-extremal black holes. It turns out, however, that the Wald entropy formula [2], applicable to general diffeomorphism invariant theories, agrees with other approaches where they are applicable. A derivation for the Wald entropy formula in the context of Euclidean quantum field theory has been given in [3] for general diffeomorphism invariant theories. It is however desirable to understand the emergence of this general formula for the black hole entropy also from a more fundamental theory of quantum gravity.

Loop quantum gravity (LQG) [4, 5] has emerged as a candidate theory for quantum gravity and addressed the question of black hole entropy with considerable success. See [6] for a review. In LQG, one has been mainly interested in black hole entropy calculations for four-dimensional GR with minimally coupled matter fields. However, it was shown that for a non-minimally coupled scalar field, the black hole entropy can be obtained with the right dependence on the scalar field's value at the horizon [7, 8, 9], in accordance with the Wald formula. While this agreement presented an important confirmation for the robustness of the LQG framework, its proper origin remained obscure.

In this paper, we will show how the Wald entropy formula naturally arises in LQG based on the recently introduced dimension-independent connection variables. The main idea is that for a generalized theory of gravity, such as Lovelock gravity or a non-minimally coupled scalar field, the direct analog of the area operator, which is a key ingredient in the entropy calculation,

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does not measure the area, but the Wald entropy. The reason is that the fluxes conjugate to the connection are given by derivatives of the Lagrangian with respect to certain curvature components, in the same way as in the construction of Wald entropy. We present higher-dimensional Lovelock gravity as an explicit example. We shortly comment on general diffeomorphism invariant theories, where no robust general statements can be made from the LQG perspective at the moment. Our current understanding thus remains restricted to theories which can be formulated on the phase space of higher dimensional GR coupled to standard matter fields. Also, we restrict to theories which do not have additional constraints on top of the Hamiltonian and spatial diffeomorphism constraints of GR, plus additional gauge constraints or the simplicity constraint discussed below. Such additional constraints may require special treatment on top of the existing techniques in the LQG entropy calculations. Since an explicit solution for them might not be available, their effect on the black hole entropy would remain an open question. The class of treatable theories thus includes Lovelock gravity [10] with non-minimal couplings of scalar fields, plus additional minimally coupled matter fields (the independence of the entropy on standard minimally-coupled matter fields is already well-understood [11]).

The paper is organized as follows. In section 2, we review some background material concerning the Wald entropy formula, as well as the black hole entropy calculations in the isolated horizon framework in LQG. Next, we show in section 3 that the entropy calculation for higher-dimensional Lovelock gravity can be reduced to that of higher-dimensional GR. We explain how this result can be understood in general in terms of the Wald entropy formula. The prospects for general diffeomorphism-invariant theories are discussed in section 4. In section 5, we comment on the interpretation of these results. Section 6 contains the conclusions of the paper.

## 2 Preliminaries

We begin with a brief review of the relevant fields. In section 2.1, we review the broad status of black hole entropy in generalized gravity theories. In sections 2.2-2.3, we review the central features of the existing black hole entropy calculations in LQG.

### 2.1 Wald entropy

The black hole entropy formula for general diff-invariant theories of gravity was first proposed in [2] and expanded on in [12, 13]. Previously, an entropy for the Lovelock class of theories was derived in [14]. The defining property which motivated the original proposal [2] was the First Law of thermodynamics for stationary black holes.

A key concept in the entropy's derivation was that of a Noether potential [15]. For every local gauge symmetry of a field theory, there exists a Noether potential  $Q^{\mu\nu}$ . This is a rank-2 antisymmetric density, analogous to the Noether current  $J^\mu$  that is associated with a global symmetry. While  $J^\mu$  is integrated over a codimension-1 hypersurface to obtain a charge,  $Q^{\mu\nu}$  is integrated over a codimension-2 surface. This is a generalization of the Gauß law from electromagnetism. In fact, the Noether potential for the electromagnetic gauge symmetry is just  $\sqrt{-g}F^{\mu\nu}$ ; its integral through a codimension-2 spatial surface is the usual electric flux.

In [2], Wald used the Noether potential associated with diffeomorphisms, specifically with translations along the Killing field that becomes null at the event horizon. His insight was to view the First Law of black hole thermodynamics as a statement about the action variations under such translations. The expression for the entropy that arises from the First Law is then the integral of the Noether potential over the black hole's bifurcation surface. Now, bifurcation surfaces exist only for eternal stationary black holes. In [13], it was shown that one can substitute the bifurcation surface by an arbitrary slice of a Killing horizon. This extended the applicability

of Wald entropy to black holes that are only *currently* stationary, having been formed in a dynamical process in the past.

More explicitly, let the theory's Lagrangian be given by:

$$\mathcal{L} = \mathcal{L}(g_{\mu\nu}, R_{\mu\nu\rho\sigma}, \nabla_{\xi_1} R_{\mu\nu\rho\sigma}, \dots, \nabla_{(\xi_1} \dots \nabla_{\xi_n)} R_{\mu\nu\rho\sigma}, \psi, \nabla_{\xi_1} \psi, \dots, \nabla_{(\xi_1} \dots \nabla_{\xi_l)} \psi) \quad , \quad (2.1)$$

where  $\psi$  denotes arbitrary matter fields,  $\nabla_\mu$  is the covariant derivative associated with the metric  $g_{\mu\nu}$ , and  $R_{\mu\nu\rho\sigma}$  is its Riemann curvature. Let  $D + 1$  be the spacetime dimension. The Wald entropy is then given by [12]:

$$S = -2\pi \int_H \sqrt{h} \frac{\delta \mathcal{L}}{\delta R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu} \epsilon_{\rho\sigma} d^{D-1}x \quad . \quad (2.2)$$

Here,  $H$  is a slice of the horizon,  $\sqrt{h}$  is its area density,  $\epsilon_{\mu\nu}$  is its normal bivector with  $\epsilon_{\mu\nu} \epsilon^{\mu\nu} = -2$ , and  $\delta \mathcal{L} / \delta R_{\mu\nu\rho\sigma}$  is the variational derivative of the Lagrangian with respect to  $R_{\mu\nu\rho\sigma}$ :

$$\begin{aligned} \frac{\delta \mathcal{L}}{\delta R_{\mu\nu\rho\sigma}} &:= \frac{\partial \mathcal{L}}{\partial R_{\mu\nu\rho\sigma}} - \nabla_{\xi_1} \left( \frac{\partial \mathcal{L}}{\partial \nabla_{\xi_1} R_{\mu\nu\rho\sigma}} \right) + \dots \\ &+ (-1)^n \nabla_{\xi_1} \dots \nabla_{\xi_n} \left( \frac{\partial \mathcal{L}}{\partial \nabla_{(\xi_1} \dots \nabla_{\xi_n)} R_{\mu\nu\rho\sigma}} \right) \quad . \end{aligned} \quad (2.3)$$

In particular, for GR, one has:

$$\mathcal{L} = \frac{1}{16\pi G} R; \quad \frac{\delta \mathcal{L}}{\delta R_{\mu\nu\rho\sigma}} = \frac{1}{32\pi G} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}); \quad S = \frac{1}{4G} \int_H \sqrt{h} d^{D-1}x = \frac{A_H}{4G} \quad . \quad (2.4)$$

The entropy formula (2.2) was also recovered in an analysis of the Euclidean black hole action [3], along the lines of the Gibbons-Hawking derivation [16] for GR. A key difficulty was to properly handle the boundary contributions to the action, without the guidance of a standard variational principle which for GR leads to the York-Gibbons-Hawking boundary term. The solution was to notice that the conserved charges fix the boundary contribution at infinity, while at the bifurcation surface a nonvanishing contribution can only come from the extrinsic curvature.

Also notable is the detailed agreement between microscopic calculations of black hole entropy within string theory (first performed in [17]) and the Wald formula. See e.g. [18]. In this context, one applies the Wald formula to the effective supergravity action, with higher-derivative terms (tightly constrained by supersymmetry) arising from stringy effects. In some cases, the relevant part of the effective action can be found exactly, and the comparison between the Wald and microscopic entropies carried out to all orders in an asymptotic expansion [19].

A major open question is how to define Wald entropy for non-stationary horizons. While the Bekenstein-Hawking area law (2.4) for GR extends unambiguously to a general horizon slice, that is not true for the more complicated formula (2.2). This issue has been partially addressed in [12]. See also [20] for a discussion in the context of black hole hydrodynamics. For Lovelock gravity, the imaginary action calculation in [21], as well as its Euclidean counterpart [22], favor a particular non-stationary definition that depends only on the intrinsic geometry of the horizon slice. This result is in agreement with the more general proposal in [12].

Other issues concern the positivity and increase properties of Wald entropy. Unlike the area law, the positivity of (2.2) may be field-dependent. This calls into question the general interpretation of (2.2) as a statistical entropy. Partial evidence suggests that negative Wald entropy is associated with pathological theories or solutions. In [23], it was shown that for a simple  $f(R)$  theory, the positivity of Wald entropy is related to cosmic censorship. Also, in

[24], it was suggested that the Wald formula can be viewed as the ordinary Bekenstein-Hawking entropy, but with the Newton's constant read off from the propagator of area perturbations on the horizon. This implies that the entropy's positivity may be correlated with the stability of the black hole.

Current understanding of entropy increase, i.e. the Second Law of thermodynamics, is again partial. In GR, the area increase theorem implies that the Second Law holds for very general processes. In general diff-invariant theories, it was shown [25] that for *quasi-stationary* accretion of matter, the Second Law is an automatic consequence of energy positivity. Little is known regarding non-stationary processes. One hope is that the requirement of entropy increase can serve to fix the correct *definition* of entropy for non-stationary horizons. This has only been achieved [25] for  $f(R)$  theories, which are equivalent to GR with a non-minimally coupled scalar. In black hole hydrodynamics (a mildly non-stationary regime), the Second Law is expressed as a non-negative viscosity. In [26], black hole hydrodynamics in Lovelock gravity was studied for planar black holes in 4+1d AdS space. It was shown there that whenever the AdS background exists, the viscosity is indeed non-negative. Finally, as possible evidence *against* the Second Law, see the argument in [27] regarding entropy decrease in Lovelock gravity during black hole mergers.

## 2.2 LQG entropy calculations in 3+1d

The LQG calculation of black hole entropy has been originally performed in 3 + 1 dimensions in terms of the Ashtekar-Barbero variables. The essential idea is that the canonical transformation from the ADM phase space [28] to Ashtekar-Barbero variables [29, 30] yields a boundary contribution in the form of a Chern-Simons symplectic potential on the isolated horizon. The original calculations [11, 31, 32, 33] were performed using a gauge fixing of SU(2) to U(1) on the isolated horizon. It was later shown that the calculation could be performed without this gauge fixing in a manifestly SU(2)-invariant manner [34, 35]. The recent introduction of a higher-dimensional and supersymmetric generalization of loop quantum gravity [36, 37, 38, 39, 40, 41, 42] made it necessary to reconsider these calculations and extend them to the new framework [43]. These results will be summarized in section 2.3. In the present subsection, we review the state of affairs in the original LQG framework, where the quantum aspects are already well-developed. However, the generalization to higher dimensions should go through similarly [43].

The basic idea of the entropy calculation is as follows. First, one derives from classical GR a boundary condition on the isolated horizon, along with a Chern-Simons symplectic potential. Upon quantization, this gives a quantum Chern-Simons theory describing the horizon degrees of freedom. In particular, the total area of the horizon is related to the Chern-Simons degrees of freedom by the quantum boundary condition. The dimension of the Chern-Simons Hilbert space, constrained to a fixed value of the total area, yields an entropy of the form:

$$S = \frac{\alpha A_H}{\gamma G} . \quad (2.5)$$

Here,  $\alpha$  is some numerical constant, while  $\gamma$  is the Barbero-Immirzi parameter, a free parameter of the theory. It is tempting to set  $\gamma = 4\alpha$  in order to obtain the well-known Bekenstein-Hawking entropy  $A_H/4G$ . However, this approach is probably too naive [44]. We defer this issue to section 5. In relation to the constant factor in (2.5), we note the work [45]. There, the correct entropy  $A_H/4G$  was obtained by setting  $\gamma = \pm i$  in the formula for the Chern-Simons Hilbert space dimension, working in the large-spin limit with a fixed number of punctures. We remark that the area proportionality of the entropy is already a non-trivial result. In particular, it depends on using the correct combinatorics for the punctures, which follows from a proper study of the action of the diffeomorphism group.

Any derivation of black hole entropy should be tested on gravity theories with dynamics other than GR. Simple examples include Lovelock gravity [10], as well as GR with non-minimally coupled matter, such as a conformally coupled scalar. It has been shown [7, 8] that for the conformally coupled scalar, LQG produces the correct Wald-entropy analog of eq. (2.5). Specifically, one performs an LQG quantization using the Ashtekar-Barbero connection and its canonical conjugate (which is no longer the usual area flux). The standard isolated-horizon calculation then leads to the entropy formula:

$$S = \frac{\alpha A_H}{\gamma G} a(\phi); \quad a(\phi) = 1 - \frac{1}{6} \phi^2. \quad (2.6)$$

This is the Wald entropy for the conformally coupled scalar, up to the same constant factor as in (2.5). As shown in [9], one can again apply the  $\gamma \rightarrow \pm i$  procedure for large spins to get the correct entropy including its constant factor.

In this paper, we aim to place the results (2.5)-(2.6) in context, as well as to generalize them to higher-derivative gravity theories. We should note at this point that the result (2.6), as well as our generalization of it, holds only for a particular choice of quantization variables. In particular, the fundamental connection variable is taken to have the same geometric meaning as in ordinary LQG, so that the non-minimal couplings etc. affect only its conjugate flux. In section 5, we will discuss the implications and interpretation of different choices of variables. We will also comment there on how one should compare the LQG results for the entropy to the semiclassical Bekenstein-Hawking-Wald formula.

### 2.3 Entropy in higher-dimensional LQG

In this paper, we consider an arbitrary even<sup>1</sup> number  $D + 1 \geq 4$  of spacetime dimensions. Up to the recent work [43], all papers on entropy calculations within LQG have been based on Ashtekar-Barbero or self-dual Ashtekar variables. In this subsection, we will briefly introduce the higher-dimensional connection variables, and review the entropy results [43] that arise from them. The current status is that the classical part of the LQG entropy calculations has been generalized to even spacetime dimensions, and a possible quantization has been sketched. However, an explicit proper quantization is still missing, and we thus have to *assume* that the usual quantization of isolated horizons as pioneered in [33] goes through. It seems to us that this is a safe assumption, since the structure of the degrees of freedom is very similar to the one in four dimensions.

In short [46], General Relativity and supergravity in  $D + 1$  dimensions can be rewritten in terms of an  $\text{SO}(D + 1)$  Yang-Mills phase space. The conjugate variables are an  $\text{SO}(D + 1)$  connection  $A_{aIJ}$  and a densitized generalized hybrid vielbein  $\pi^{aIJ}$ , related to the spatial metric via  $2qq^{ab} = \pi^{aIJ}\pi^b_{IJ}$ , where  $q = \det q_{cd}$ . Here,  $a, b = 1, \dots, D$  are spatial tensor indices on the spatial slice  $\sigma$  in the  $D + 1$  decomposition of spacetime.  $I, J = 0, \dots, D$  are internal indices transforming under  $\text{SO}(D + 1)$ . In addition to the usual constraints of GR - a Hamiltonian constraint, a spatial diffeomorphism constraint and in our case an  $\text{SO}(D + 1)$  Gauß constraint - one must also introduce a so-called simplicity constraint. This constraint, given by  $\pi^a_{[IJ}\pi^b_{KL]} \approx 0$ , ensures<sup>2</sup> that  $\pi^a_{IJ} = 2n_{[I}E^a_{J]}$ . Here,  $n^I$  is a unit normal defined by  $E^a_I n^I = 0$ , while  $E^a_I$  coincides in the “time gauge”  $n^I = (1, 0, \dots, 0)$  with the densitized  $D$ -bein derived from a spatial metric

<sup>1</sup>In odd dimensions, the boundary symplectic structure cannot be rewritten in Chern-Simons form. However, it was suggested in [43] that this might be in fact unnecessary, and that one can use  $\sqrt{\hbar} \times n^{[I} s^{J]}$  as the boundary degrees of freedom for all  $D \geq 2$ . Our arguments for obtaining the leading term in the entropy then work in the same way for both even and odd dimensions. Nevertheless, for simplicity, we will restrict to even dimensions for the rest of this paper.

<sup>2</sup>In four dimensions, a topological sector appears and the situation is more complicated, see [36].

$q_{ab}$ , i.e.  $E_i^a E_j^b \delta^{ij} = qq^{ab}$ ,  $i, j = 1, \dots, D$ . Furthermore, an additional rescaling by a free real parameter  $\beta$  takes place, so that the momentum becomes  $^{(\beta)}\pi_{KL}^a := \pi_{KL}^a / \beta$ . This  $\beta$  is analogous to, but different from the Barbero-Immirzi parameter  $\gamma$ . In the quantum theory, the simplicity constraint can be implemented<sup>3</sup> on the links of a spin-network by restricting the representations of  $\text{SO}(D+1)$  to be of class 1, so that their highest weight vector  $\vec{\lambda}$  is determined by a single non-negative integer  $\lambda$  as  $\vec{\lambda} = (\lambda, 0, \dots, 0)$  [47].

As in the standard isolated-horizon calculations (section 2.2), the canonical transformation to the variables above leads to a boundary term in the symplectic potential. As usual, we take the boundary  $H = \partial\sigma$  of our hypersurface to be a horizon slice. Let us denote the area density and the Euler density on  $H$  by  $\sqrt{h}$  and  $E^{(D-1)}$ , respectively. Now, restrict to a part of phase space where the scalar ratio  $E^{(D-1)}/\sqrt{h}$  is constant (this is referred to in [43] as the non-distortion condition; we will see in section 3 that it can be lifted). This condition implies in particular that up to a numerical factor,  $E^{(D-1)}/\sqrt{h}$  is the same as  $\chi/A_H$ , where  $\chi$  is the Euler characteristic of  $H$ . The second variation of the boundary symplectic potential can then be rewritten in terms of a Chern-Simons symplectic structure as [43]:

$$\delta_{[1} \int_{\sigma} \partial_a \left( \frac{1}{\beta} E^{aI} \delta_{2]} n_I \right) d^D x = \text{const} \times \frac{A_H}{\beta \chi} \int_H \text{Tr}_{\epsilon} [\delta_{[1} \Gamma^0 \wedge \delta_{2]} \Gamma^0 \wedge R^0 \wedge \dots \wedge R^0] . \quad (2.7)$$

Here,  $\text{Tr}_{\epsilon}[X_1 X_2 \dots X_{(D+1)/2}] := X_1^{IJ} X_2^{KL} \dots X_{(D+1)/2}^{MN} \epsilon_{IJKL\dots MN}$ ,  $\Gamma^0$  is a generalization of the Peldan hybrid connection [48] defined on  $H$ , and  $R^0$  is the curvature of  $\Gamma^0$ . Furthermore, one can derive the boundary condition:

$$\epsilon^{IJ\dots KLMN} \epsilon^{\alpha\beta\dots\delta\sigma} R_{\alpha\beta IJ}^0 \dots R_{\delta\sigma KL}^0 = \text{const} \times \frac{\beta \chi}{A_H} \times \hat{s}_a^{(\beta)} \pi^{aMN} , \quad (2.8)$$

where  $\alpha, \beta = 1, \dots, D-1$  are tensor indices on  $H$ ,  $\hat{s}_a \equiv \epsilon_{a\beta\gamma\dots\sigma} \epsilon^{\beta\gamma\dots\sigma} / (D-1)!$  is the (appropriately densitized) normal covector to  $H$  in  $\sigma$ , and  $^{(\beta)}\pi^{aMN} := \pi^{aMN} / \beta$ . It seems that in higher dimensions, a stronger form of the boundary condition is necessary to avoid having local degrees of freedom on the horizon [43]. We will not discuss these details here, as they are not important for the arguments presented in this paper.

### 3 Entropy in Lovelock gravity from LQG

Lovelock gravity [10] is the most general higher-derivative theory of pure gravity that has no more than two time derivatives, so that it can be formulated on the same phase space as (higher-dimensional) GR. Up to a boundary term given in [49], the Lovelock action reads:

$$S = \int_{\mathcal{M}} d^{D+1}x \sqrt{-g} \mathcal{L} = \int_{\mathcal{M}} d^{D+1}x \sqrt{-g} \sum_{m=0}^{\lfloor \frac{D+1}{2} \rfloor} c_m \mathcal{L}_m , \quad (3.1)$$

where  $c_m$  are coupling constants, e.g.  $c_1 = 1/(16\pi G)$ , and:

$$\mathcal{L}_m = \frac{(2m)!}{2^m} R^{[\mu_1 \nu_1} R^{\mu_2 \nu_2} \dots R^{\mu_m \nu_m]}_{[\mu_1 \nu_1 \mu_2 \nu_2 \dots \mu_m \nu_m]} . \quad (3.2)$$

The canonical formulation of this theory can be developed in analogy to the well-known ADM treatment [28], and is given in [50]. There is a certain problem in the analysis, since the extrinsic curvature cannot be expressed uniquely in terms of the metric and its canonical conjugate.

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<sup>3</sup>See [38, 40] for a discussion of possible anomalies and the implementation on a node.



This can lead to a multivalued Hamiltonian constraint. However, this does not seem to be of direct concern to us, since the constraint algebra and the spatial diffeomorphism constraint are unaffected [50]. The Hamiltonian constraint does not enter the calculation, since the lapse function vanishes at the horizon [11]. Maximally symmetric black hole solutions of the type considered in [43] have been discussed in [51] for higher-dimensional Lovelock gravity. The thermodynamics of Lovelock gravity has first been studied in [52], where it is shown that the entropy acquires a non-trivial prefactor depending on the coupling constants and the Riemann curvature of the horizon slice.

Our goal is to derive the canonical variables of Lovelock gravity that are relevant for the entropy calculation, and to relate them to the corresponding canonical variables in pure GR. It is more instructive to use the first-order Palatini formulation of the theory, since we want to calculate the momentum conjugate to the connection. Interestingly, Lovelock gravity is the only higher-derivative generalization of GR for which the first order Palatini-type action agrees with the second order Einstein-Hilbert-type action [53].

In first-order form, the Lovelock Lagrangian reads:

$$\mathcal{L} = - \sum_{m=0}^{\lfloor \frac{D+1}{2} \rfloor} c_m \frac{(2m)!}{2^m} e_{I_1}^{[\mu_1} e_{J_1}^{\nu_1} \dots e_{I_m}^{\mu_m} e_{J_m}^{\nu_m]} F_{\mu_1 \nu_1}^{I_1 J_1} F_{\mu_2 \nu_2}^{I_2 J_2} \dots F_{\mu_m \nu_m}^{I_m J_m}, \quad (3.3)$$

where  $F_{\mu\nu}^{IJ}$  is the curvature of the  $\text{SO}(1, D)$  connection  $A_\mu^{IJ}$ . The minus sign is chosen to agree with the conventions in [37]. The gauge group here is not  $\text{SO}(D+1)$ , since we are starting from a covariant framework. The transition from  $\text{SO}(1, D)$  to  $\text{SO}(D+1)$  as an internal gauge group while maintaining the Lorentzian signature of spacetime is detailed in [36, 37], and involves a canonical transformation. The reason that this trick works is based on the fact that both sets of connection variables are related via phase space reductions to the ADM phase space, which coincides for Euclidean and Lorentzian signature. The signature of spacetime is encoded in the Hamiltonian constraint of the theory as a relative sign between two terms. To avoid confusion, we will keep  $\text{SO}(1, D)$  as the internal gauge group for the rest of this paper. The transition to  $\text{SO}(D+1)$  would only change some signs.

The canonical momentum conjugate to  $A_a^{IJ}$  reads:

$$\begin{aligned} \pi_{KL}^a &= e \frac{\partial \mathcal{L}}{\partial \dot{A}_a^{KL}} = -2\sqrt{q} n_\mu \frac{\partial \mathcal{L}}{\partial F_{\mu a}^{KL}} \\ &= 4\sqrt{q} n_\mu \sum_{m=1}^{\lfloor \frac{D+1}{2} \rfloor} m \frac{(2m)!}{2^m} c_m e_K^{[\mu} e_L^a e_{I_2}^{b_2} e_{J_2}^{c_2} \dots e_{I_m}^{b_m} e_{J_m}^{c_m]} F_{b_2 c_2}^{I_2 J_2} \dots F_{b_m c_m}^{I_m J_m}. \end{aligned} \quad (3.4)$$

For  $m=1$ , we obtain the usual momentum  $\pi_{IJ}^a = 2n_{[I} E_{J]}^a$  with  $E_J^a := \sqrt{q} e_J^a$ , familiar from the canonical analysis of the higher-dimensional Palatini action [37]. For consistency with [37], we set  $8\pi G = 1$ , i.e.  $c_1 = 1/2$ .

We are interested in the effect of the new canonical momentum (3.4) on the black hole calculations. Thus, in analogy to [7, 8, 9], we must rewrite the boundary condition and the symplectic structure in terms of the new momentum. Here, several simplifications arise. We are only interested in the canonical momentum on the horizon, and only in its  $\hat{s}_a \pi_{IJ}^a$  component (recall that  $\hat{s}_a$  is an appropriately densitized normal to  $H$  within the hypersurface  $\sigma$ ). This

component reads:

$$\begin{aligned}\hat{s}_a \pi_{KL}^a &= -\sqrt{h} \epsilon_{\mu\nu} \frac{\partial \mathcal{L}}{\partial F_{\mu\nu}^{KL}} \\ &= 2\sqrt{h} \epsilon_{\mu\nu} \sum_{m=1}^{\lfloor \frac{D+1}{2} \rfloor} m \frac{(2m)!}{2^m} c_m e_K^{[\mu} e_L^{\nu]} e_{I_2}^{\alpha_2} e_{J_2}^{\beta_2} \dots e_{I_m}^{\alpha_m} e_{J_m}^{\beta_m} F_{\alpha_2 \beta_2}^{I_2 J_2} \dots F_{\alpha_m \beta_m}^{I_m J_m} .\end{aligned}\quad (3.5)$$

We see that all the field strengths in (3.5) are pulled back to  $H$ . It can be shown [43] that this pullback equals  $F_{\alpha\beta IJ} = R_{\alpha\beta IJ}^0 = {}^{(D-1)}R_{\alpha\beta}{}^{\gamma\delta} e_{\gamma I} e_{\delta J}$ , where  ${}^{(D-1)}R_{\alpha\beta\gamma\delta}$  is the Riemann tensor of the intrinsic metric on  $H$ . It then follows that the  $KL$  indices in (3.5) must lie in the plane of the binormal  $\epsilon_{KL} = 2n_{[K} s_{L]}$ . We get:

$$\begin{aligned}\hat{s}_a \pi_{KL}^a|_H &= \frac{1}{2} \sqrt{h} \epsilon_{KL} \frac{\partial \mathcal{L}}{\partial F_{\mu\nu}^{IJ}} \epsilon_{\mu\nu} \epsilon^{IJ} \\ &= \sqrt{h} \epsilon_{KL} \times \sum_{m=1}^{\lfloor \frac{D+1}{2} \rfloor} 2m c_m \frac{(2m-2)!}{2^{m-1}} {}^{(D-1)}R_{[\alpha_2 \beta_2} [\dots {}^{(D-1)}R_{\gamma_m \delta_m]} \gamma_m \delta_m] \\ &= -\sqrt{h} \epsilon_{KL} \frac{\partial \mathcal{L}}{\partial R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu} \epsilon_{\rho\sigma} \\ &=: \sqrt{\tilde{h}} \epsilon_{KL} .\end{aligned}\quad (3.6)$$

where  $\epsilon_{\mu\nu} = 2n_{[\mu} s_{\nu]}$  is the binormal from section 2.1. Note that the equality between the first and third lines on the RHS of (3.6) follows from the equivalence between the first-order and second-order Lovelock actions. The factor 1/2 between the first and the third line comes from different conventions for the definition of the derivative used in the literature: for the first line, we are consistent with [37], while for the third line, we are consistent with [2]. Again, the minus sign between these lines comes from the sign choice [37] for the first-order Lagrangian.

We see that  $\hat{s}_a \pi_{KL}^a$  basically measures not the area density  $\sqrt{h}$ , but the density  $\sqrt{\tilde{h}}$  of Wald entropy (in units of  $1/4G = 2\pi$ ). For the conformally coupled scalar field [7, 8, 9], we would have a similar result, with:

$$\hat{s}_a \pi_{KL}^a = \sqrt{\tilde{h}} \epsilon_{KL} = -\sqrt{h} \epsilon_{KL} \frac{\partial \mathcal{L}}{\partial R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu} \epsilon_{\rho\sigma} = \sqrt{h} \epsilon_{KL} \left(1 - \frac{\phi^2}{6}\right) . \quad (3.7)$$

Coming back to Lovelock theory, it remains to rewrite the boundary symplectic structure and the boundary condition from section 2.3 using the Lovelock conjugate variables. We then find that (2.7) becomes:

$$\delta_{[1} \int_{\sigma} \partial_a \left( \frac{1}{\beta} \tilde{E}^{aI} \delta_{2]} n_I \right) d^D x = \text{const} \times \frac{\tilde{A}_H}{\chi \beta} \int_H \text{Tr}_{\epsilon} \left[ \delta_{[1} \tilde{\Gamma}^0 \wedge \delta_{2]} \tilde{\Gamma}^0 \wedge \tilde{R}^0 \wedge \dots \wedge \tilde{R}^0 \right] , \quad (3.8)$$

Here,  $\tilde{A}_H$  is the “area” given by the integral of  $\sqrt{\tilde{h}}$ , i.e. the Wald entropy in units of  $1/4G = 2\pi$ . The connection  $\tilde{\Gamma}^0$  with curvature  $\tilde{R}^0$ , which in [43] were built from the horizon metric, can now be built from *any* metric<sup>4</sup> whose “area” density is  $\sqrt{\tilde{h}}$ . As in [43], the only additional restriction<sup>5</sup>

<sup>4</sup>More precisely, given a metric  $\tilde{h}_{\alpha\beta}$  satisfying  $\det(\tilde{h}_{\alpha\beta}) = \tilde{h}$ , we construct a  $D+1$ -bein  $\tilde{e}_{\alpha}^I$  such that  $\tilde{h}_{\alpha\beta} = \tilde{e}_{\alpha}^I \tilde{e}_{\beta}^I$  and  $\tilde{e}_{\alpha}^I n_I = \tilde{e}_{\alpha}^I s_I = 0$ . Then,  $\tilde{\Gamma}^0 = \Gamma^0(\tilde{e})$ .

<sup>5</sup>It seems that a metric satisfying these two requirements can always be found for suitable horizon topologies: take a spherically symmetric metric  $h_{ab}^s$  on  $H$ . Pick a diffeomorphism  $\Phi$  on  $H$  such that  $\sqrt{\tilde{h}} = \Phi^*(\sqrt{\det h_{\alpha\beta}^s})$ . Then,  $\tilde{h}_{\alpha\beta} := \Phi^*(h_{\alpha\beta}^s)$  also satisfies the non-distortion condition, since this condition is a scalar. Thus, the non-distortion condition does *not* pose any restriction on the actual spacetime metric. Still,  $\chi = 0$  would lead to ill-defined expressions and we exclude that case from the Chern-Simons treatment. See however footnote 1.



on this metric is that the associated Euler density  $\tilde{E}^{(D-1)}$  satisfies  $\tilde{E}^{(D-1)}/\sqrt{h} = \text{const}$  on  $H$ . As for the boundary condition (2.8), it becomes:

$$\epsilon^{IJ\dots KLMN}\epsilon^{\alpha\beta\dots\delta\sigma}\tilde{R}_{\alpha\beta IJ}^0\dots\tilde{R}_{\delta\sigma KL}^0 = \text{const} \times \frac{\chi^\beta}{\tilde{A}_H} \times \hat{s}_a^{(\beta)}\pi^{aMN}. \quad (3.9)$$

In the same manner as already observed in [7, 8, 9] for the conformally coupled scalar field, the computation of the entropy now works analogously to the pure-GR case. The only conceptual difference is that we are now counting states which correspond not to a given macroscopic area  $A_H$ , but to a given value of  $\tilde{A}_H$ . Indeed, the analog of the area operator built from the Lovelock fluxes has the familiar discrete spectrum [38]  $\text{const} \times \sqrt{\lambda(\lambda + D - 2)}$ ,  $\lambda \in \mathbb{N}_0$ , but measures  $\tilde{A}_H$  rather than  $A_H$ . Using a straightforward generalization of the techniques developed in [54] to calculate the entropy (neglecting logarithmic corrections), one arrives at:

$$S_{\text{Lovelock}} = \frac{\tilde{\alpha}\tilde{A}_H}{\beta G}, \quad (3.10)$$

which is the correct Wald entropy up to a constant coefficient.  $\tilde{\alpha}$  in (3.10) is a numerical constant analogous to  $\alpha$  in (2.5), which depends on the number of dimensions.

We remark that the properties of isolated horizons used in [43] remain valid in Lovelock gravity (and indeed in any theory), since they are of geometric origin and do not involve the field equations.

At this point, it becomes apparent why the Wald entropy formula and the LQG black hole entropy calculations agree: the generalized area operator is constructed roughly as  $\tilde{A} \sim \sqrt{\text{flux}^2}$ . The flux variables  $^{(\beta)}\pi^{aKL}$  conjugate to the connection are not measuring the (internal bivector-valued) area, but the derivative of the Lagrangian with respect to the curvature tensor component  $R_{\mu\nu\rho\sigma}\epsilon^{\mu\nu}\epsilon^{\rho\sigma} \sim R_{nsns}$  as in the Wald entropy formula. Here, the first  $n$  index comes from the time derivative of the connection when defining the conjugate momentum. The first  $s$  index comes from the  $s_a$ -component of the momentum that's relevant for the entropy calculation. The second  $n$  and  $s$  result from the fact that for Lovelock gravity plus non-minimally coupled scalars, only the internal  $s^{[InJ]}$ -component is non-vanishing. Whether this last statement is true in more general situations is presently unclear.

## 4 The prospects for general diff-invariant theories

It was shown in [3] that general diffeomorphism-invariant theories with Lagrangian of the form (2.1) can be rewritten as a higher-dimensional gravity theory with no higher derivatives, coupled to additional (partially symmetric) tensor fields. Essentially, the additional degrees of freedom resulting from the higher time derivatives are traded for these tensor fields. In this process, new equations of motion which relate the tensor fields to derivatives of the Riemann tensor have to be imposed via Lagrange multipliers. In the canonical formalism, this translates into additional constraints.

These results are an important step towards treating general diff-invariant theories within LQG, in the manner illustrated in section 3. However, there remain several problems that prevent us from making any solid statements about LQG black hole entropy calculations for such theories.

1. The canonical  $D + 1$  decomposition of symmetric tensor fields leads to additional terms proportional to the extrinsic curvature in the split action, and the calculation of the canonical conjugate to the extrinsic curvature becomes complicated. The expression of the split

action given in [3] hints at the conjugate of the extrinsic curvature being related to (2.3), thus potentially leading to an LQG derivation of the proper Wald entropy. However, this relies on treating the metric and extrinsic curvature as independent variables, which comes at the cost of additional second-class constraints. Before quantizing, one must solve these second-class constraints classically. After doing so, the symplectic structure may again turn out to be too complicated.

2. Symmetric tensor fields have not been treated so far by LQG methods (unlike  $p$ -forms - see [42]). It seems that a construction similar to the connection variables for the metric might be necessary, which could yield additional boundary degrees of freedom. These degrees of freedom could contribute to the entropy.
3. After quantizing, one must take into account any leftover first-class constraints. While the lapse function and thus the smeared Hamiltonian constraint vanish at the horizon, it is not clear what effects the additional first-class constraints might have on the entropy.

The situation for general diffeomorphism invariant theories is thus rather unclear at the moment. It seems that the best way to proceed is to study simple examples on a case by case basis to get a better feeling for them. We leave this for further research.

## 5 Interpretation and the choice of quantization variables

In the above, we've generalized the LQG entropy results (2.5)-(2.6) to higher-derivative theories of gravity. We found that if one quantizes using the ordinary LQG connection (in the version appropriate to arbitrary dimensions, but otherwise retaining its geometric meaning) and its conjugate flux (which loses its simple geometric meaning), then the Wald entropy is recovered up to a constant factor. While this is an interesting result, it represents only a step towards a full understanding of black hole entropy within LQG. The caveats that must be raised appear already in the more familiar setting of LQG with Ashtekar-Barbero variables in 3+1d. Since more is known about that setting, we will use it for the purpose of the discussion. We expect that our comments below will also be relevant to the dimension-independent setup of sections 3-4.

### 5.1 Naive interpretation

Let us begin with the standard LQG result (2.5) for GR with minimally coupled matter. The naive response to this result is to set  $\gamma = 4\alpha$ , thus recovering by “brute force” the correct numerical coefficient for the Bekenstein-Hawking formula  $S = A_H/4G$ . Now, the Barbero-Immirzi parameter  $\gamma$  (like the analogous parameter  $\beta$  in the arbitrary-dimensional setup) defines a family of different quantization choices. Each is associated with a choice of fundamental connection variable, which is to be subjected to the LQG quantization procedure. Thus, the naive interpretation of (2.5) would be that there is a single preferred choice of quantization variables.

For GR with a conformally coupled scalar, this naive conclusion becomes sharper. One has now a larger selection of plausible connection variables to quantize. In particular, the constant parameter  $\gamma$  can be replaced with a function of the scalar field  $\phi$ . Two choices appear especially natural. One choice, adopted in [7, 8], is to maintain the standard meaning of the fundamental connection, at the cost of its conjugate flux no longer measuring area. This is the choice described in section 2.2 and the direct analog of the choice adopted by us in sections 3-4. It leads to the

correct Wald entropy up to a constant, as shown in eq. (2.6). Another choice<sup>6</sup> [9] is to maintain the geometric meaning of the fundamental flux as a measure of areas, at the cost of changing the meaning of the connection. This leads to the GR entropy formula (2.5) instead of (2.6), i.e. gives a wrong functional dependence of the entropy on the scalar field. Thus, again the naive conclusion is that there is a single preferred choice of quantization variables that produces the correct Wald entropy:

1. One must maintain the geometric meaning of the connection rather than the flux. This fixes the quantization variables up to a constant  $\gamma$  and gives the correct Wald entropy up to a constant.
2. Then, as in GR, one must fix further  $\gamma = 4\alpha$ .

As we will now explain, this interpretation is in fact unfounded. On the other hand, a modified version of it appears to hold for large spins (see section 5.3).

## 5.2 Semiclassical limits and the continuum

The argument in section 5.1 is missing a crucial ingredient. One must always keep in mind that the Bekenstein-Hawking formula refers to a semiclassical regime of gravity. For instance, the Newton's constant  $G$  appearing there comes from the prefactor of the semiclassical action. For Wald's generalization of the entropy formula, the same remark applies. Thus, any comparison of the LQG entropy to the Bekenstein-Hawking-Wald result must be in the context of some semiclassical limit. See also [44] for a discussion.

As discussed in [55], there are two semiclassical limits that one may consider in LQG. One is the limit of continuum GR, which is supposed to emerge from LQG states with very many spins and intertwiners. The other limit consists of coherent states with very large spins. This is a special subclass of states in the theory, characterized by large quantum numbers and large discrete elements of geometry. The large-spin limit is thus quite distinct from the continuum one. (We leave open the possibility that the continuum limit may *also* be described in terms of large spins, through some notion of coarse-graining and RG flow. However, such coarse-grained degrees of freedom would then have an effective dynamics distinct from that of the fundamental theory.)

The continuum classical limit (if it exists) is of course the main focus of physical interest. It is also the domain of the LQG entropy formula (2.5), since the latter receives contributions mainly from many small spins. However, we have no independent knowledge about the effective action in the continuum. For instance, if one takes the coarse-graining approach, one does not know how to relate the coarse-grained dynamics to that of the fundamental theory. In particular, the relation between the fundamental Newton's constant (appearing in the entropy result (2.5)) and the effective Newton's constant in the continuum (appearing in the Bekenstein-Hawking formula) is unknown.

Thus, there is no direct conclusion that can be drawn from eqs. (2.5)-(2.6) or from our generalization (3.10). It may be that, as argued in section 5.1, there is a unique quantization that correctly produces the continuum limit. Or it may be that all quantizations are equally good, and the results for the entropy are all correct when reexpressed in terms of the effective continuum action.

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<sup>6</sup>In [9], on top of using the physical metric in the choice of canonical variables, a constant mean curvature gauge fixing of the Hamiltonian constraint was employed. This leads to fluxes rescaled by the scalar field, but by a different function from the one appearing in the Wald entropy. We will neglect this detail in this paper, as it is not important here, and the calculation in [9] can also be done without this gauge fixing.

On the other hand, if one examines the large-spin semiclassical limit, a very different picture emerges, with a conclusion quite similar (but not identical) to that of section 5.1. This picture is the result of several strands of recent work, culminating in [55]. We now turn to describe it.

### 5.3 The large-spin limit and sending $\gamma$ to $\pm i$

In the large-spin semiclassical limit, the state of knowledge concerning the dynamics is much better. In particular, we have at our disposal an effective action derived from spinfoam amplitudes. The one analyzed in the greatest detail is the 4-simplex vertex amplitude [56]. Now, it was shown in [55] that this amplitude reproduces the correct classical GR action only if one sets  $\gamma = \pm i$  at the end of the calculation<sup>7</sup>. This is contrary to previous claims that the correct action is reproduced for any real  $\gamma$ . The conclusion of [55] rests on the recent observation [57, 21] that the classical GR action has an imaginary part. A full agreement between the spinfoam amplitude and the classical action, *including* the imaginary part, is obtained if and only if one sets  $\gamma = \pm i$ . We note that effective action of [56, 55] has the same Newton’s constant as the fundamental theory. This is an expected result for coherent states with large quantum numbers, which need not hold for the continuum.

Now, as discussed in section 2.2, the recent LQG entropy calculation [45] shows that restricting to a fixed number of large spins and *then* sending  $\gamma$  to  $\pm i$  results in the correct Bekenstein-Hawking entropy, including the constant factor. It was shown in [9] that the same procedure also works for GR with a conformally coupled scalar, provided that one starts with the quantization variables from [7, 8], i.e. doesn’t alter the geometric meaning of the connection<sup>8</sup>.

To sum up, the large-spin limit seems to describe a semiclassical regime only if one retains the geometric interpretation of the connection and sets  $\gamma = \pm i$  after the quantum calculations have been performed. This procedure produces both a correct semiclassical action (i.e. a correct relation between its real and imaginary parts) and a correct black hole entropy (i.e. a correct relation between the entropy and the semiclassical action).

We expect that similar statements should also hold for higher-dimensional Lovelock gravity. However, to be certain, one must compare the entropy result to an effective action derived from a spinfoam amplitude. Writing a spinfoam model for generalized theories of gravity and calculating its large-spin behavior is a non-trivial task, and we will not comment on it further in this paper.

## 6 Conclusion

Working with the dimension-independent connection variables, we’ve related the LQG black hole entropy calculation to the Wald entropy formula. The key point is that the generalized area operators measure a rescaled version of the area at the horizon, which is essentially the Wald entropy. The reason for this is that the variable conjugate to the connection along the  $\hat{s}_a$  direction (the spacelike horizon normal) is given by the derivative of the Lagrangian with

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<sup>7</sup>This corresponds to self-dual connection variables. Of course, it is not known how to define the quantum theory for non-real  $\gamma$ . Thus, the closest one can get to a quantum theory based on self-dual variables is to send  $\gamma \rightarrow \pm i$  *after* the quantum calculation with real  $\gamma$ . Real  $\gamma$  can thus be seen as a regulator.

<sup>8</sup>It was also shown in [9] that if one starts from different variables, retaining the geometric meaning of the *flux* (as explained in footnote 6, we neglect here the constant mean curvature gauge fixing in [9] for simplicity), one can recover naively the correct entropy formula by sending an *effective* Barbero-Immirzi parameter  $a(\phi)\gamma$  to  $\pm i$ . However, it’s not clear how to define this procedure in any context other than an entropy calculation for a homogeneous horizon. In particular, it’s not clear how to apply it to an effective action calculation, which is necessary for a proper comparison to the Bekenstein-Hawking formula. This is the reason for the “naively” qualifier above.

respect to the curvature component  $R_{nsns}$ . The same quantity, integrated over the horizon slice  $H$ , enters the Wald entropy formula. Our analysis has covered non-minimally coupled scalars and Lovelock gravity.

The main open problem for the entropy calculation is the comparison with semiclassical actions. In [55], this has been done for four-dimensional pure gravity in a “transplanckian” large-spin regime. However, in the continuum, the problem remains open. The same is true even for large spins in higher dimensions, as well as for non-minimally coupled matter or Lovelock gravity, due to the lack of a corresponding spin foam model. Also, a quantization of the isolated horizon degrees of freedom has not yet been developed in detail for the dimension-independent variables; see [43] for a discussion on first steps. As discussed in section 4, general diff-invariant theories are not yet under control. Reliable conjectures about their entropy as derived from loop quantum gravity, if such a calculation exists at all, cannot be made presently.

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